MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

PCL0016 – CALCULUS

(ALL Sections/Groups)

8 MARCH 2016
9:00 A.M - 11:00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 5 pages including cover page with SIX questions.
2. Attempt **ALL SIX** questions and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.

Question 1 (20 marks)

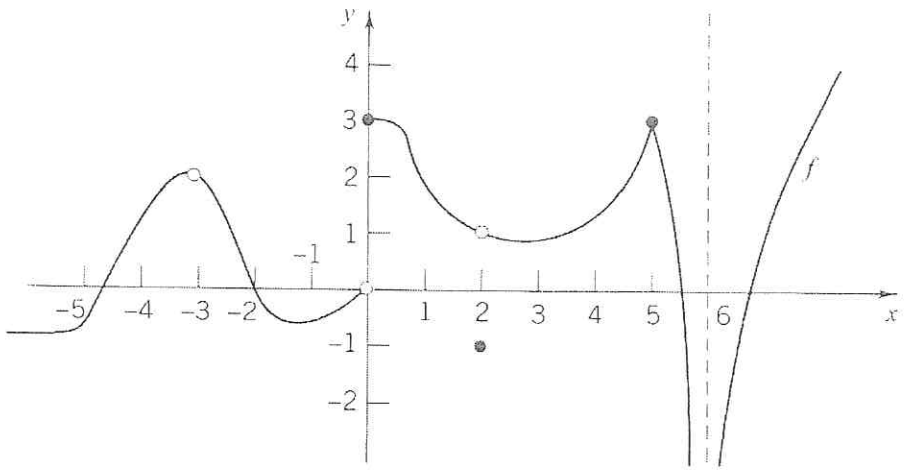

Figure 1

(a) Given a function $f(x)$ whose graph is shown in **Figure 1**, determine the values of the following limits and justify your reason.

(i) $\lim_{x\to 0} f(x)$ [4 marks]

(ii) $\lim_{x\to 2} f(x)$ [4 marks]

(b) Given $f(x)=2x^2-4x$, find $f'(x)$ using its definition $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

[4 marks]

(c) Evaluate the following limits. (**Do not use** L'Hôpital's Rule):

(i) $\lim_{x\to 0}\frac{\sec(x)-1}{x}$ [4 marks]

(ii) $\lim_{x\to 3}\frac{\sqrt{x+6}-3}{x-3}$ [4 marks]

Continued.....

Question 2 (15 marks)

(a) Find the equation of the tangent line to the curve $f(x) = 2x^3 - 4x^2 + 2x - 1$ at the point where $x = 2$. [6 marks]

(b) Given the function $y = 3\sec^4(2x)$, find the exact value of $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}}$. [5 marks]

(c) Given $x^2y^2 + x\sin y = 3$, find $\frac{dy}{dx}$ by implicit differentiation. [4 marks]

Question 3 (15 marks)

(a) Given a function $f(x) = x^3 + 3x^2 - 45x$, identify the interval(s) on which the function is increasing or decreasing. [4 marks]

(b) A rock thrown upward from the ground with an initial velocity of 60 m/s is $s(t) = 60t - 12t^2$ meters above the ground t sec after it has been thrown. This equation is valid from the time the ball is thrown until the time it returns to the ground.

(i) Calculate the velocity and acceleration of the rock at any time t. [2 marks]
(ii) Calculate the duration it takes for the rock to reach its highest point. [1 mark]
(iii) Determine maximum height the rock can go. [1 mark]
(iv) Determine the duration it takes for the rock to reach the ground. [2 marks]
(v) Determine the velocity of the rock when it hits the ground. [1 mark]

(c) As shown in **Figure 2**, a rectangular storage area is to be constructed along the side of a tall building. A security fence is required along the remaining 3 sides of the area with 1000 m of fencing. What is the length x that can provide maximum area? [4 marks]

Figure 2

Continued.....

Question 4 (15 marks)

(a) Find:

(i) $\int \cos(x+1)(2+\sin(x+1))^{100}\,dx$ using integration by substitution.

[3 marks]

(ii) $\int \frac{x}{e^{2x}}\,dx$ using integration by parts.

[3 marks]

(iii) $\int_3^4 \frac{\ln(5x-5)}{x-1}\,dx$ using definite integration by substitution. [3 marks]

(b) Given the rational function

$$f(x)=\frac{16}{x^3-2x^2}.$$

use partial fraction decomposition to show that it can be written in the form

$$f(x)=-\frac{4}{x}-\frac{8}{x^2}+\frac{4}{x-2}.$$

[4 marks]

Hence, determine $\int f(x)dx$.

[2 marks]

Question 5 (15 marks)

A region Y is bounded by $y=x^2-9x$ and $y=81-x^2$ as shown in **Figure 3**.

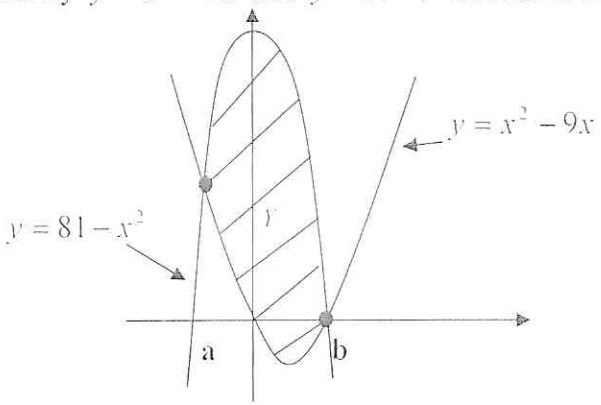

Figure 3

(a) Find the points of intersection at a and b. [3 marks]

(b) Find the area of the bounded region Y. [6 marks]

(c) Find the volume generated when region Y is rotated about $x=10$ using the cylindrical shell method. [6 marks]

Continued.....

Question 6 (20 marks)

(a) Show that $y = A\sin 2x + B\cos 2x$ is a general solution to the differential equation $\frac{d^2y}{dx^2} = -4y$. [5 marks]

(b) Solve the differential equation $2x^2 y\frac{dy}{dx} = (1+x^2)(1+y^2)$ given that $y(1) = 1$. Leave the answer in the form of $y^2 = f(x)$. [11 marks]

(c) Find the general solution of $y'' - 42y' + 441y = 0$. [4 marks]

End of Paper